QUANTUM HALL EFFECT AS A RESISTANCE STANDARD

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ABSTRACT

In a two-dimensional electron gas system, the Hall resistance at low temperatures and in strong magnetic field has plateaus as a function of the number of electrons that can be linked to natural constants as $R_H = h/ie^2$. Due to the high precision of the measurement, the quantization of the Hall resistance is now used as the primary standard of resistance. The paper describes the use of quantum Hall effect as a mean to calibrate a reference standard of 1 kΩ dc resistance having a relative uncertainty of a few parts in 10^8.

INTRODUCTION

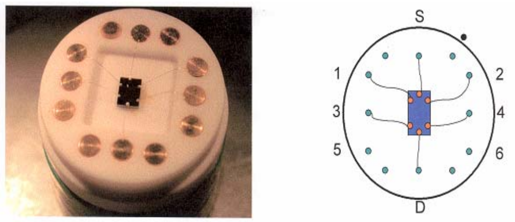
The quantum Hall effect (QHE) provides an invariant reference for resistance linked to the natural constants, 'h' the Plank constant and 'e' the electronic charge [1,2]. The quantum Hall effect (QHE) which is a characteristic of a two-dimensional electron gas (2DEG) provides an invariant reference for resistance linked to the natural constants, 'h' the Plank constant and 'e' the electronic charge [1-2]. The 2DEG can-- be realized in a high mobility semiconductor device such as silicon MOSFET (metal-oxide-semiconductor field effect transistor) or GaAs-$Al_xGa_{1-x}As$ heterostructure, In GaAs Hall device, the 2DEG is located in the inversion layer formed at the interface between two semiconductors, one of them acting as the insulator. When the Hall device is cooled to a low temperature (~1K) and subjected to high applied flux density (6-9 T), the 2DEG is completely quantized and, for the fixed source to drain current, the Hall voltage V_H shows a series of constant voltage plateaus (called Hall plateaus) as a function of the applied field. In the limit of zero dissipation, that is, the longitudinal resistance along the direction of the current remains zero over the plateau region, the Hall resistance of the i^{th} plateau $R_H(i)$ is quantized according to the relation: $R_H\ (i) = V_H/I_{SD} = h/ie^2 = R_K/i$. Here 'i' is an integer and R_K is the von Klitzing constant. It follows that for i =1 $R_H(1) = R_K$. The QHE has a universal character, i.e., it does not depend on the (1) device type, (2) plateau index, (3) mobility of the charge carriers (4) device width at the level of the relative uncertainty of 3×10^{-10} [3] The R_K = 25812.807 Ω was adopted by all member states of the Meter Convention and came into effect as of 1st January 1990. The resistance scaling process linking the QHR to decade value resistors uses Cryogenic Current Comparator (CCC), Josephson potentiometer, Hamon network or Direct Current Comparator (DCC) bridge. The smallest uncertainty can be obtained using a CCC and a 100 Ω standard resistor.

In this paper, we report the calibration of 1kΩ Tinsley standard resistor against QHR using a DCC bridge from 18^0C to 26^0C, maintained in an oil enclosure. The standard resistor is used in a stirred and thermo-stated oil bath that holds the temperature to within 0.02 ^{0}C.

EXPERIMENTAL

The quantum Hall device employed is a GaAs-AlGaAs heterostructure with annealed tin ball contacts at the edges of the rectangular bar. Contacts on the narrow edges acts as current leads and the other four in two pairs opposing each other along the longer edges for potential contacts. The sample is mounted on a ceramic TO8 header and the ball contacts are bonded to the header pins using 20 mil gold wire. The device and the schematic are illustrated in Fig. 1.

The QHR device is plugged into a socket on the end of a probe. The probe is inserted through an O-ring at the top of the cryostat, and adjusted vertically to the center of the superconducting magnet. The probe also carries a calibrated ruthenium oxide temperature sensor and a heater. The probe is isolated from the helium reservoir by a double walled tube, the space between the two walls being evacuated to isolate the inner sample space from ^{4}He reservoir. Impedance connects inner space with the ^{4}He reservoir. Pumping on this inner space draws ^{4}He into the tube and reduces the temperature of the inner space to 1.2 K. The superconducting magnet has a bore diameter just greater than the inner wall tube. A current of 56 A at 4.2 K produces an induction of 9 T in the center region. The field at the center has ± 0.1% homogeneity over 1 cm^2. The magnetic field can be put into persistent mode when required, to conserve helium.



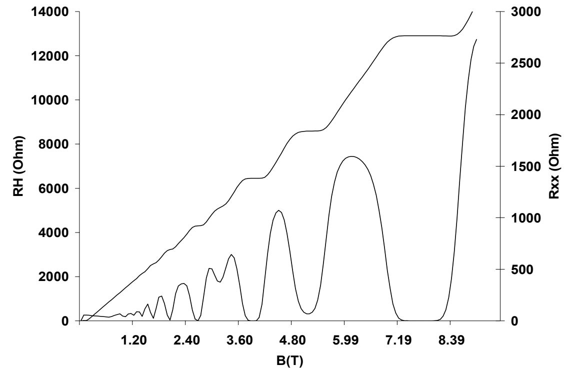


Fig. 1(left). Quantum Hall device mounted on a TO8 header and schematic of the contacts, Figure 2(right) Longitudinal and Hall resistance as a function of the magnetic field.

The DCC bridge (6010Q) being used presently has a binary step-up of the windings that allows a self-calibration of the current ratio. An accuracy of a few parts in 10^{-8} can be achieved for the measured ratio of 13:1 or 10:1. The accuracy of ratio measurement is limited by the current and voltage noise associated to the current comparator and the nV detector. The DCC bridge capable of performing the sweep test of the Hall and longitudinal resistances, dissipation and three terminal contact resistances, is interfaced to a PC via IOTECH personal 488 interface card and drivers. Figure 2 (right) show the field sweep for QOhm0303 at ~1.2 K for one set of pair. The Hall resistance $R_H = R_{xy} = V_{xy}/I_{SD}$ and the longitudinal resistance R_{xx} are measured at source-drain current I_{SD} of 77 μA. Wide and well-quantized i = 2 plateau at magnetic flux in the range 7.2 T to 7.9 T is observed. The quantized plateau of the R_H occurs where the R_{xx} vs B vanishes and satisfies $R_H = h/ie^2$ to give a constant value of resistance over the range of its flatness.

Since large contact resistance in a QHR device can lead to erroneous values due to improper quantization therefore accurate determination of all the contact resistances is unavoidable. The contact resistances are measured at the center of i = 2 plateau with the superconducting magnet set to persistent mode. The source and drain contact resistance are measured at 77 μA while others at 10 μA. The values of the resistances obtained were less than 1Ω provide the device is cooled slowly from room temperature to the working temperature. Dissipation in the longitudinal voltage V_{xx} is made using 6010Q in the nano-volt mode. The field is swept over the plateau i = 2 and V_{xx} recorded.

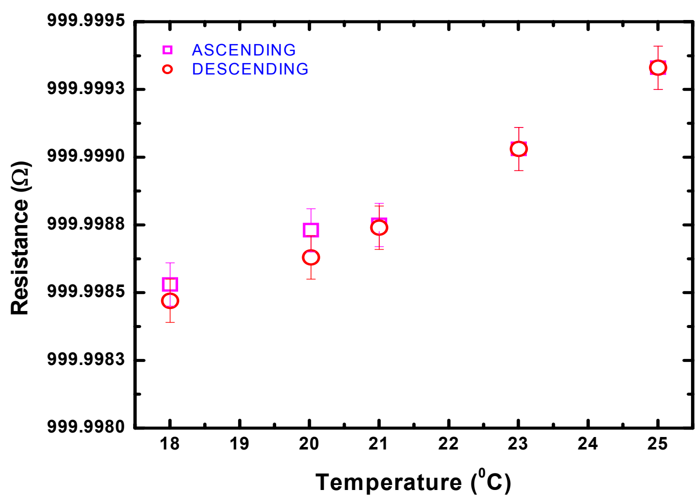


Fig.3: Variation of secondary standard resistor with bath temperature

Finally the secondary resistor (of nominal value 1000Ω) maintained in a constant temperature oil enclosure is measured against the QHR. The temperature of the resistor is continuously monitored using calibrated thermometers. Figure 3 shows the value of the 1kΩ Tinsley 5685B, Serial No. 279594, resistor, measured against QHR value of 12906.4035 Ω at i = 2, at different temperature along with the associated uncertainties. Six sets of 50 readings were taken for the measurement and out of which last 35 are chosen for statistical analysis. The 1 kΩ resistor was calibrated in the temperature range 18 – 25 ^{0}C with ascending and descending order of temperature. This was carried out in order to observe the effect of thermal cycling and the stability of the resistor. As seen in the figure, when the temperature is 21 ^{0}C or above the resistor values overlap while in the lower temperature range there is a small difference in the values measured during the ascending and descending cycles. This shows that the resistor has much better stability at temperatures ~23 ^{0}C. The deviation from the values is however in the uncertainty range of our calibrations.

Evaluation of Uncertainty in Measurement

As an example we discuss uncertainty evaluation for one particular resistance value. The mathematical model used to evaluate the value of the unknown resistor is as follows:

$$R_x = (R_s + \delta R_S)R_R^{-1}(R_{nlb}) + c_x\delta t_x + \Delta R_x \qquad (1)$$

Where, R_x is value of the unknown resistor, R_S is value of the reference (QHR) resistor, δR_S relative uncertainty of the primary resistor, R_R (= R_S/R_X) ratio of the reference resistor and the unknown resistor, R_{nlb} is non-linearity and instability of the DCC bridge, δt_x is temperature drift of the unknown resistor, c_x is temperature coefficient of the unknown resistor and ΔR_x is correction to R_x. The combined standard uncertainty (u_C) is given by the expression

$$u_C^{\ 2}(R_X) = c_1^{\ 2}u_1^{\ 2}(\delta R_S) + c_2^{\ 2}u_2^{\ 2}(R_R) + c_3^{\ 2}u_3^{\ 2}(R_X) + c_4^{\ 2}u_4^{\ 2}(R_{NLB}) + c_5^{\ 2}u_5^{\ 2}(\delta t_x) + c_6^{\ 2}u_6^{\ 2} \qquad (2)$$

Here c_1, c_2, c_3, c_4, c_5 and c_6 are the sensitivity coefficients and u_1, u_2, u_3, u_4, u_5 and u_6 are the standard uncertainties of the input estimates.

Type A Evaluation

Standard Uncertainty Due to Repeatability in R_x

The standard uncertainty in the data is taken as the standard derivation of mean of 210 readings. In this case the measured standard uncertainty u_3 = 4.3603 x 10^{-6}Ω. The degree of freedom ν_1= 209 and the sensitivity coefficient c_1 = 1.

Standard Uncertainty in Ratio (R_R)

The standard uncertainty in the ratio was evaluated to be u_2 = 6.56522 x 10^{-8}. The degree of freedom is ν_2= 209 and the sensitivity coefficient is c_2 = 1Ω.

The Type B Evaluation

Relative uncertainty of the quantum Hall resistor

The quantum Hall device and DCC bridge was calibrated at National Research Council of Canada using CCC and the relative uncertainty of the quantum Hall resistor is reported to be better than 1 x 10^{-8}. Assuming a rectangular distribution the standard uncertainty in the QHR is u_3 = 0.57735 x 10^{-8} and the sensitivity coefficient is c_3 = 1 Ω with the degree of freedom $\nu_1 = \infty$.

Standard Uncertainty due to non-linearity and instability of the DCC

It is mentioned in the calibration certificate issued by National Research Council of Canada that for 13:1 calibration the combined expanded uncertainty at k=2 is 2.7 x 10^{-5} Ω. Therefore, assuming a normal distribution, the standard uncertainty due to the non-linearity and instability of the DCC is $u_4 = 1.35 \times 10^{-5}\Omega$. The sensitivity coefficient is $c_4 = 1$ and the degree of freedom is $\nu_4 = \infty$.

The uncertainty due to temperature drift of 1 kΩ resistor

The temperature drift of the oil bath is estimated to be ± 0.02°C. Assuming a rectangular distribution one has $u_5 = 0.02/\sqrt{3} = 0.011547$. The temperature coefficient (sensitivity coefficient) of the unknown resistor as mentioned in the manufacturer's specification sheet is $2.0\text{x } 10^{-3}\Omega$ /°C. Thus the sensitivity coefficient $c_5 = 2.0\text{x } 10^{-3}\Omega$ /°C with degree of freedom being $\nu_5 = \infty$. We also take into account the uncertainty of the thermometer used to monitor the oil bath temperature. According to the calibration certificate of the thermometer the combined standard uncertainty at k = 2 is 0.024 °C. Therefore, assuming a normal distribution, the standard uncertainty in temperature measurement is $u_6 = 0.024$ °C /2.0 = 0.012 °C. In this case too, $c_6 = 2.0\text{x } 10^{-3}\Omega$ /°C and the degree of freedom is $\nu_6 = \infty$.

Combined Standard uncertainty

Combined Standard uncertainty (u_c) is calculated by using the value of individual standard uncertainties and sensitivity coefficients in equation (2). Thus

u_c^2 (Rx) = $1.3106 \times 10^{-09}\Omega$ and hence $u_c = 3.62022 \times 10^{-5}\ \Omega$

Effective degree of freedom

Using the values of u_c and various u_i and ν_i we get the effective degree of freedom is calculated to be $\nu_e = 3.1202\text{x}10^{+5}$.

Expanded Uncertainty

Since $\nu_e > 100$, at 95% confidence level the coverage factor is k = 2. Thus the expanded uncertainty is $U = k{*}U_c (R_x) = 7.24044 \times 10^{-5}\ \Omega$ or 0.0724 ppm.

CONCLUSION

With DCC the comparison of 1kΩ resistor against QHR has been carried out in the temperature range 18^0C to 25^0C with ascending and descending order of temperature. This shows that the resistor has much better stability at temperatures ~23 ^{0}C. The expanded uncertainty of the measurements is ~ 0.08 ppm and can be further improved by using high precision thermometer as the bath stability is ~0.001^0C.

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