Measuring the Quantum Hall Resistance using a room temperature DC current comparator bridge

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Abstract: We have compared a 100 Ω wire resistor with a QHR (step $i = 2$) using a cryogenic current comparator bridge and using a commercial dc room temperature current comparator bridge. In the latter case the measurement was made via an intermediary (1 kΩ) wire resistor. The difference between the values obtained for the 100 Ω resistor using the two bridges is $- 0.3 \pm 1.4$ $\mu\Omega$, or $- 0.003 \pm 0.014$ ppm.

1. Introduction

Quantum Hall resistors (QHRs) have a defined value (R_H) of 25,812.807 Ω on step $i = 1$, with appropriate sub-multiples of this value on other steps. Such resistors are used as representations of the ohm in the national laboratories of many countries, where it is common practice to compare these primary resistance standards on a regular basis with a set of thermally stabilised wire resistors. Such comparisons are in many cases carried out using cryogenic current comparator bridges (CCCB) that in the best cases can routinely be used to obtain values for nominally 100 Ω wire resistors with both resolution and repeatability of a few hundred nano-ohms. A binary CCCB can be designed to be self-checking by an appropriate choice of numbers of turns for the set of coils used, and the consistency of the measurement system as a whole can be checked by measuring various different ratios of resistors. Hence one can obtain a value of the 100 Ω resistor in terms of R_H with an accuracy of 1 $\mu\Omega$ or better.

Nevertheless there are difficulties associated with implementing a CCCB, amongst these the necessity to cool the coil with liquid helium. Consequently other methods have been, and in some cases still are, used to compare QHR resistors with wire resistors [for eaxmple 1,2,3]. We have previously suggested the use of a dc room temperature current comparator bridge (dcCCB) for such a measurement [4]. According to the manufacturer this bridge has accuracy < 0.2 ppm in the range between 10 kΩ and 100 kΩ, and a linearity of 0.01 ppm. The ratio accuracy and linearity of such a bridge was tested at the National Physical Laboratory in the UK (NPL), where it was shown that the dcCCB tested was in agreement with the NPL CCCB to better than 0.02 ppm, across the range 1 Ω to 10 kΩ [5]. In this note we report the recent use of a dcCCB to determine the value of a 100 Ω wire resistor with respect to step $i = 2$ of a QHR resistor. The difference between the results from the two bridges is less than 1 $\mu\Omega$, or less than 0.01 ppm.

2. Measurement procedure

In the Electrical Standards group at the Institute for National Measurement Standards we regularly calibrate dcCC bridges against our CCCB. Typical calibration data obtained for ratios of decade values from 1Ω to 10 kΩ, using a set of wire resistors maintained at constant temperature ($\pm$ 50 mK) in an oil bath, are shown in Figure 1. The differences between the dcCCB and the NRC CCCB, are shown as diamonds, and the error bars are the combined uncertainties (95% confidence) of the two measurement systems being compared. Following the calibration of this particular bridge[1], with the agreement of the manufacturer, we kept the unit at NRC for an extra week to carry out the measurements reported here.

With the CCCB we typically measure from the QHR i = 2 value of 12,906.4035 Ω directly to a 100 Ω wire resistor, using a coil ratio of 129:1. Commercial dcCCBs are typically optimised for scaling ratios of approximately 10:1, or 13:1 in some cases. To allow a comparison with the CCCB it was necessary to carry out the measurements with the dcCCB in two stages. A first step from the QHR i = 2 value to a 1000 Ω wire resistor, using the 13:l ratio was followed by a second step from 1000 Ω to 100 Ω using the 10:1 ratio.

A GaAs/AlGaAs heterostructure device (#NRC1794B) was cooled to 0.3 K in a ^{3}He refrigerator and it was kept cold throughout the measurement period. This device is a rectangular chip from an NRC fabricated wafer [6], with six tin-ball contacts annealed at the edges, two on the narrower ends to act as current leads and the other four in two pairs opposing each other along the longer edges for potential contacts. At 7.5 T the device is at the centre of the i = 2 plateau. Using the CCCB to compare the Hall resistance of each of the two pairs of potential contacts with a Tinsley 100 Ω resistor showed agreement between the two pairs better than 2 x 10^{-9}, with a standard error of $\pm$ 3 x 10^{-9}. Dissipation along the device in the rectangle defined by the potential contacts was negligible, as measured with a nanovoltmeter (EM-N11). The current used for all these checks was 50 μA, the same current as was used for the device in all measurements reported here.

The procedure for the comparison of the two bridges, which was completed in one twenty-four hour period, was as shown in Table 1, along with some measurement details. The 1 kΩ and 100 Ω resistors used (both Guildline type 9330) were maintained in an oilbath at 25.00 $\pm$ 0.05 °C. The mid-time of the third measurement, the dcCCB comparison of the 1 kΩ and 100 Ω resistors, is taken as the reference time, t_{ref}, for the comparison, and values of each of the ratios, at that time, are determined by interpolation.
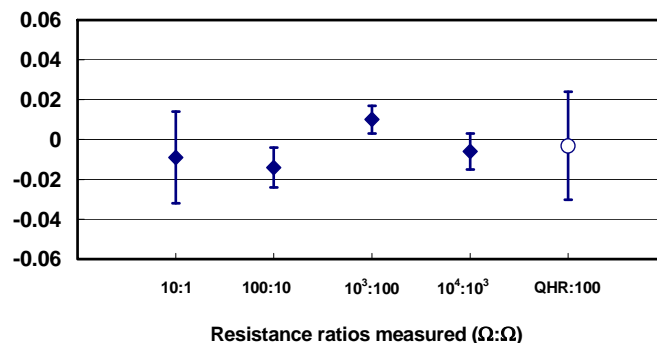

Figure 1: Difference in ratio value for the two bridges, in ppm deviation from nominal, expressed as (dcCCB – CCCB). The uncertainties are combined values from the two bridges, and are approximately 95% confidence levels.

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[1] The unit tested was a Measurements International Ltd. type 6010Q dc Current Comparator Bridge.

3. Results

By combining the results of measurements 2, 3 and 4 in Table 1 we find the deviation of the QHR:100 Ω ratio as determined by the dcCCB to be -13.734 ± 0.013 ppm. The difference between the dcCCB and CCCB values for this ratio is consequently -0.003 ± 0.014 ppm. This is equivalent to a difference of -0.3 $\mu\Omega \pm 1.4$ $\mu\Omega$ between the two bridges in determining the value of the 100 Ω resistor assuming the QHR to be the defined 12 906.4035 Ω. This datum is shown on Figure 1as the circular point. The plotted error bars are the uncertainty given in Table 1, which is determined by the quadrature combination of the uncertainties of the dcCCB and CCCB ratios.

The symmetry of the procedure eliminates uncertainties due to drift in the wire resistors during the measurement: and with the currents as shown, there are no power effects to take into account. Any resistance changes as a consequence of variations in temperature of the oil bath will be accounted for in the noise of the measured values. In the case of the dcCCB values in Table 1 the uncertainties are calculated by combining the Type A uncertainty from each
measurement with an appropriate additional term for the scatter of the individual points.

Type A uncertainties in the CCC measurement values, which include factors such as the detected voltage noise and the uncertainty of the measured values of the coil ratios, are included in the

	Bridge	Resistors compared	Resistor currents (mA)	No. of data points	Ratio at t_{ref}	Uncertainty (ppm)
1	CCCB	QHR – 100 Ω	0.05 – 6.45	1	-13.731	.004
2	dcCCB	QHR – 1 kΩ	0.05 – 0.65	3	-14.770	.023
3	dcCCB	1 kΩ – 100 Ω	0.65 – 6.50	6	1.036	.003
4	dcCCB	QHR – 1 kΩ	0.05 – 0.65	3	-14.770	.023
5	CCCB	QHR – 100 Ω	0.05 – 6.45	1	-13.731	.004

Table 1: Measurement sequence used in the comparison of the two bridges. The ratios in column 6 are determined at time t_{ref}, the mid-time of measurement 3, by interpolation. Uncertainties are in each case the quadrature sum of the uncertainties of the two measurement systems (95% confidence).

uncertainty with which we know each of the CCC data points. All other ("Type B") uncertainties in the CCC measurements arise from the trim calibration, from leakage between the current sources, and from the input bias current and the output non-linearity of the EM N11 nanovoltmeter used as detector. The total contribution from these factors is always small (typically < 0.004 ppm) and varies slightly with ratio. The Type B uncertainty value has been summed in quadrature with the Type A values determined as in the paragraph above.

4. Conclusion

We have described the comparison of a 100 Ω wire resistor with a QHR (step $i = 2$) using a cryogenic current comparator bridge and using a commercial dc room temperature current comparator bridge. We find a difference between the values obtained for the 100 Ω resistor using the two bridges is $- 0.3 \pm 1.4$ $\mu\Omega$, or $- 0.003 \pm 0.014$ ppm.

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